

# Logics of Deontic Inconsistencies and Paradoxes

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## Abstract

Usually, a deontic paradox consists of a set of sentences in natural language which are, intuitively, logically independent and jointly consistent but, when formalized in standard deontic logic (**SDL**), the set derives contradictory obligations and/or has logical dependencies. In this paper, following the approach initiated in [8], two deontic systems based on *Logics Formal Inconsistency* are proposed: the first one is closer to **SDL** because contradictory obligations trivialize but contradictory sentences do not. The second one is a bimodal extension of system **DmbC** introduced in [8] by adding a modality for deontic inconsistency as a primitive, such that contradictory obligations do not trivialize. This approach overcomes deontic paradoxes such as Chisholm's paradox by allowing a richer repertoire of connectives which avoids logical dependencies, and by avoiding logical collapse in the presence of contradictory obligations.

## 1 Introduction

Traditional logics have difficulty to deal with contradictions. The problem, in general, resides in the so-called *Principle of Explosion (PE)* (see [3]):

$$(PE) \quad \forall\Gamma\forall\alpha\forall\beta(\Gamma, \alpha, \neg\alpha \vdash \beta)$$

The Logics of Formal Inconsistency (**LFI**'s), introduced in [4] (see also [3]), are logics that allow to deal with contradictions by internalizing the notion of consistency and inconsistency by means of connectives  $\circ$  and  $\bullet$ , respectively. These logics do not respect (PE), but a weak version of (PE): a contradiction  $\alpha$  and  $\neg\alpha$  is not, in general, explosive, unless the consistency  $\circ\alpha$  of  $\alpha$  is also assumed. This idea is formalized by the Weak Principle of Explosion (WPE):

$$(WPE) \quad \forall\Gamma\forall\alpha\forall\beta(\Gamma, \alpha, \neg\alpha, \circ\alpha \vdash \beta)$$

In the same sense, Standard Deontic Logic (**SDL**) has difficulty to deal with conflicting obligations ( $\bigcirc\alpha$  and  $\bigcirc\neg\alpha$ ) and this property generates several deontic paradoxes, such as *contrary-to-duties*. Until now, many alternative systems

to **SDL** were proposed in order to avoid paradoxes. Since the notion of consistency can be internalized in **LFI**, may be useful “externalize” that notion in deontic logic by the Principle of Deontic Explosion (PDE):

$$(PDE) \quad \forall \Gamma \forall \alpha \forall \beta (\Gamma, \bigcirc \alpha, \bigcirc \neg \alpha \vdash \bigcirc \beta)$$

But, if inconsistency has to be controlled in a deontic paradox, it would be necessary to get a weaker version of (PDE), by requiring that “ $\alpha$  is deontically consistent”. In formal terms, we would have the following principle (WPDE):

$$(WPDE) \quad \forall \Gamma \forall \alpha \forall \beta (\Gamma, \bigcirc \alpha, \bigcirc \neg \alpha, \bigcirc \circ \alpha \vdash \bigcirc \beta)$$

In [8] two deontic systems were proposed such that (PDE) does not hold, but (WPDE) holds good. That logics were called **LDI**'s. It is worth noting that in [9] a paraconsistent deontic system was introduced, but under a different approach than [8]. A recent approach to modal **LFI**'s, closer to **LDI**'s, can be found in [1].

In this paper, the notion of deontic inconsistency makes clear by a closer analysis of the principles above exposed. Two new systems, **SDmbC** and **BDmbC**, are proposed. The first one is interesting because separates the notions of **LFI** and **LDI**. The second one is a bimodal system having two deontic operators, one that respects (PDE) and the other respecting (WPDE). As done in [8], the well-known deontic paradox called *Chisholm Paradox* will be analyzed; however, in the light of the new system **BDmbC**, much more tools are available in order to understand the paradox and dissolve it.

## 2 SDL and DmbC

In order to have a new perspective of the conflicting obligation problem, in [8] was proposed a variant of **SDL** called **DmbC** in which, inspired by **LFI**'s, the notions of “consistency” and “deontic consistency” can be internalized. The first step is to consider a suitable axiomatization of **SDL**:<sup>1</sup>

**Definition 1** The Standard Deontic Logic **SDL** is defined over the signature  $\{\wedge, \vee, \Rightarrow, \neg, \bigcirc\}$  as follows:

**Axiom Schemas:**

$$(Ax_1) \quad \alpha \Rightarrow (\beta \Rightarrow \alpha)$$

$$(Ax_2) \quad (\alpha \Rightarrow \beta) \Rightarrow ((\alpha \Rightarrow \beta \Rightarrow \gamma) \Rightarrow (\alpha \Rightarrow \gamma))$$

$$(Ax_3) \quad \alpha \Rightarrow (\beta \Rightarrow (\alpha \wedge \beta))$$

$$(Ax_4) \quad (\alpha \wedge \beta) \Rightarrow \alpha$$

$$(Ax_5) \quad (\alpha \wedge \beta) \Rightarrow \beta$$

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<sup>1</sup>For a more usual axiomatization of **SDL** see [6].

- (**Ax<sub>6</sub>**)  $\alpha \Rightarrow (\alpha \vee \beta)$   
 (**Ax<sub>7</sub>**)  $\beta \Rightarrow (\alpha \vee \beta)$   
 (**Ax<sub>8</sub>**)  $(\alpha \Rightarrow \gamma) \Rightarrow ((\beta \Rightarrow \gamma) \Rightarrow ((\alpha \vee \beta) \Rightarrow \gamma))$   
 (**Ax<sub>9</sub>**)  $\alpha \vee (\alpha \Rightarrow \beta)$   
 (**Ax<sub>10</sub>**)  $\alpha \vee \neg\alpha$   
 (**exp**)  $\alpha \Rightarrow (\neg\alpha \Rightarrow \beta)$   
 (**O-K**)  $\bigcirc(\alpha \Rightarrow \beta) \Rightarrow (\bigcirc\alpha \Rightarrow \bigcirc\beta)$   
 (**O-E**)  $\bigcirc\mathbf{f}_\alpha \Rightarrow \mathbf{f}_\alpha$  where  $\mathbf{f}_\alpha \equiv_{def} \alpha \wedge \neg\alpha$

**Inference Rules:**

- (**MP**)  $\frac{\alpha, \alpha \Rightarrow \beta}{\beta}$   
 (**O-Nec**)  $\frac{\vdash \alpha}{\vdash \bigcirc\alpha}$  ■

The axioms (Ax<sub>1</sub>) - (Ax<sub>10</sub>) and (exp) plus the rule (MP) constitute a sound and complete axiomatization of the Classical Propositional Calculus. Observe that (exp) is a way to internalize (PE) by a single axiom. On the other hand, (O-E) expresses the basic principle of **SDL** which states that contradictory obligations trivialize the system.

It is worth noting that we can consider a fragment of **SDL** excluding (O-E). That system is a deontic version of the minimal modal system **K**. On the other hand, the principle (PDE) can be derived by applying (O-Nec) and (O-K) to (exp). Any modal system that respect (O-Nec) and (O-K) is called a *normal modal system*.

As mentioned above, a **LFI** is a logic that do not respect (PE) but respect (WPE). Analogously, we say that a logic is a **LDI** whenever (PDE) do not holds but (WPDE) holds good. Clearly, **SDL** is not a **LFI** because of (exp). It is neither a **LDI** because, as observed above, it satisfies (PDE). If we want a deontic system that is a **LFI** and a **LDI** simultaneously it is necessary to replace (exp) and (O-E) by weaker versions. This is the idea behind the logic **DmbC** introduced in [8], which we reproduce below.

**Definition 2** The deontic logic **DmbC** – *Deontic mbC* – is defined over the signature  $\{\wedge, \vee, \Rightarrow, \neg, \bigcirc, \circ\}$  by substituting in **SDL** axiom schemas (exp) e (O-E) by the following, respectively:

- (**bc1**)  $\circ\alpha \Rightarrow (\alpha \Rightarrow (\neg\alpha \Rightarrow \beta))$   
 (**O-E**)<sup>◦</sup>  $\bigcirc\perp_\alpha \Rightarrow \perp_\alpha$  where  $\perp_\alpha \equiv_{def} (\alpha \wedge \neg\alpha) \wedge \circ\alpha$  ■

In other words, **DmbC** consists of axiom schemas  $(Ax_1) - (Ax_{10})$ ,  $(bc1)$ ,  $(O-K)$  and  $(O-E)^\circ$ , plus the rules  $(MP)$  and  $(O-Nec)$ .

It is worth noting that the non-modal fragment of **DmbC** (that is, **DmbC** without  $(O-K)$ ,  $(O-E)^\circ$  and  $(O-Nec)$ ) is **mbC**, the basic **LFI** introduced in [4]. It should be observed that **DmbC** is not simply the extension of **mbC** by adding the modal axioms and rules of **SDL**. In fact, **DmbC** is a deontic extension of **mbC** in which  $(O-E)$  is appropriately adapted to the paraconsistent scenario: the bottom particle  $f_\alpha$  of **SDL** (a classical one) is substituted by  $\perp_\alpha$ , the **mbC** bottom particle. This suitable version of  $(O-E)$ , called  $(O-E)^\circ$ , preserves in **DmbC** the main feature of Kripke semantics of **SDL**: the accessibility relation is serial (cf. [8] and Definition 3 below). Recalling that  $\bigcirc$  distributes over conjunctions in **DmbC** in the same way that in **SDL**, there is another way to look at  $(O-E)^\circ$ : in the presence of conflicting obligations plus the information that the formula involved is deontically consistent, the system trivializes. In particular, it follows the  $\bigcirc$ -trivialization, as expected from  $(WPDE)$ .

Note that if we add as an axiom schema that all the obligations are deontically consistent – that is,  $\bigcirc\circ\alpha$  – we recover  $(PDE)$ . Moreover, by adding  $\circ\alpha$  as axiom schema,  $(exp)$  is derived by  $(MP)$  (and so classical logic is recovered, see [4]). Then, by  $(O-Nec)$ ,  $(O-E)$  is also recovered. In that way, we may affirm that **DmbC** +  $\circ\alpha \equiv \mathbf{SDL}$ .<sup>2</sup>

In order to prove that **DmbC** is both a **LFI** and a **LDI**, it must be shown that  $(PE)$  and  $(PDE)$  *do not* hold. This is possible by using a Kripke-style semantics adequate to **DmbC**. The soundness and completeness of **DmbC** w.r.t. the semantics below was proved in [8].

**Definition 3** A *Kripke structure* to **DmbC** is a triple  $\langle W, R, \{v_w\}_{w \in W} \rangle$  such that:

1.  $W$  is a non-empty set (of *possible worlds*);
2.  $R \subseteq W \times W$  is a relation (of *accessibility*) between possible-worlds such that  $R$  is *serial* (that is, for every  $w$  there is  $w'$  such that  $wRw'$ );
3.  $\{v_w\}_{w \in W}$  is a family of mappings  $v_w : For^\circ \rightarrow \mathbf{2}$  (where  $For^\circ$  denotes the set of formulas over the signature of **DmbC**) satisfying the clauses below.
  - (i)  $v_w(\alpha \wedge \beta) = 1$  iff  $v_w(\alpha) = v_w(\beta) = 1$ ;
  - (ii)  $v_w(\alpha \vee \beta) = 0$  iff  $v_w(\alpha) = v_w(\beta) = 0$ ;
  - (iii)  $v_w(\alpha \Rightarrow \beta) = 0$  iff  $v_w(\alpha) = 1$  and  $v_w(\beta) = 0$ ;
  - (iv)  $v_w(\alpha) = 0$  implies  $v_w(\neg\alpha) = 1$ ;
  - (v)  $v_w(\alpha) = v_w(\neg\alpha)$  implies  $v_w(\circ\alpha) = 0$ ;
  - (vi)  $v_w(\bigcirc\alpha) = 1$  iff  $v_{w'}(\alpha) = 1$  for every  $w'$  in  $W$  such that  $wRw'$ . ■

<sup>2</sup>Rigorously speaking, **DmbC** +  $\circ\alpha$  is **eSDL**, the linguistic extension of **SDL** by adding the innocuous operator  $\circ$ .

Now, let  $\mathcal{M}$  be a structure as in Definition 3 such that  $W = \{w\}$  and  $wRw$ . Suppose also that  $p$  and  $q$  are propositional variables such that  $v_w(p) = v_w(\neg p) = 1$  and  $v_w(q) = 0$ . Then

$$\mathcal{M} \not\models p \Rightarrow (\neg p \Rightarrow q) \text{ and } \mathcal{M} \not\models \bigcirc p \Rightarrow (\bigcirc \neg p \Rightarrow \bigcirc q)$$

and so **DmbC** is both a **LFI** and a **LDI**.

### 3 SDmbC and BDmbC

In Section 1 we said that the inability of **SDL** to deal with conflicting obligations generates several paradoxes. But that is a very simplistic way to understand the question: axiom (O-E) is very intuitive, and so rejecting or weaken (O-E) is a way to stand back the intuitive notion of obligation behind the natural language (see [11]).

As shown in the previous section, **DmbC** considers  $(\text{O-E})^\circ$ , a different version of (O-E), and thus avoids some paradoxes. Another possibility is to consider a stronger version of  $(\text{O-E})^\circ$ , closer to (O-E), and just avoid the logical dependencies in order to overcome paradoxes. This is the aim of the system **SDmbC**.

**Definition 4** The Deontic Logic **SDmbC** – *Standard Deontic mbC* –, is obtained from **DmbC** by replacing  $(\text{O-E})^\circ$  by the following:

$$(\text{O-E})^* \quad \bigcirc \mathbf{f}_\alpha \Rightarrow \perp_\alpha \quad \blacksquare$$

Since obligations distribute over conjunctions, the axiom  $(\text{O-E})^*$  says that  $\{\bigcirc \alpha, \bigcirc \neg \alpha\}$  is enough to trivialize and so **SDmbC** is stronger than **DmbC**, validating (PDE). Thus, it is a classical deontic logic and not a **LDI**. Moreover, **SDmbC** +  $\circ\alpha \equiv \mathbf{eSDL}$

An adequate Kripke semantics for **SDmbC** is obtained from that of **DmbC** (see Definition 3) by adding the following clause:

$$(vii) \quad v_w(\bigcirc \neg \alpha) = 1 \text{ implies } v_{w'}(\alpha) = 0 \text{ for every } w' \text{ such that } wRw'.$$

In order to show that **SDmbC** is a **LFI** but not a **LDI**, consider the **SDmbC** model  $\mathcal{M}$  with  $W = \{w, w'\}$ ,  $wRw'$ ,  $w'Rw'$ ,  $v_w(p) = 1 = v_w(\neg p)$ ,  $v_{w'}(p) = 1$  and  $v_{w'}(\neg p) = 0$  (for a propositional variable  $p$ ); this invalidates (PE). The validity of (PDE) and (WPE) is a direct consequence of  $(\text{O-E})^*$  and (exp), respectively. Observe that, as a corollary, we have **DmbC**  $\subset$  **SDmbC**, that is, **SDmbC** is strictly stronger than **DmbC**.

An alternative approach to the systems above is to consider a bimodal system combining the characteristics of **DmbC** and **SDmbC**. The idea behind this is to have more possibilities to formalize a set of premises in natural language, by using a richer signature. When formalizing, it would be possible to choose a modality respecting (PDE) or not. The new system, called **BDmbC**, is axiomatized as follows:

**Definition 5** The Deontic Logic **BDmbC** – *Bimodal Deontic mbC* – is defined over the signature  $\{\wedge, \vee, \Rightarrow, \neg, \bigcirc, \boxplus, \circ\}$  by adding to **DmbC** (cf. Definition 2) the following:

**Axiom Schemas:**

$$\text{(BA)} \quad \boxplus\alpha \Rightarrow \bigcirc\alpha$$

$$\text{(\boxplus-K)} \quad \boxplus(\alpha \Rightarrow \beta) \Rightarrow (\boxplus\alpha \Rightarrow \boxplus\beta)$$

$$\text{(\boxplus-E)*} \quad \boxplus f_\alpha \Rightarrow \perp_\alpha$$

**Inference Rule:**

$$\text{(\boxplus-Nec)} \quad \frac{\vdash \alpha}{\vdash \boxplus\alpha} \quad \blacksquare$$

Intuitively,  $\boxplus$  means “classically obligatory”, “strongly obligatory” or even “consistently obligatory”. The intended interpretation of  $\bigcirc$ , by opposition, is “paraconsistently obligatory”, “weakly obligatory” or even “*prima facie* obligatory”. System **BDmbC** is not just a fusion or fibring of **DmbC** and **SDmbC**, because of the presence of the bridge axiom (BA), which relates both deontic operators.<sup>3</sup> The semantics of **BDmbC** is presented below.

**Definition 6** A *Kripke structure* for **BDmbC** is a tuple  $\langle W, R, R^\circ, \{v_w\}_{w \in W} \rangle$  such that:

1.  $W$  is a non-empty set;
2.  $R \subseteq W \times W$  and  $R^\circ \subseteq W \times W$  are serial;
3.  $R \subseteq R^\circ$ ;
4.  $\{v_w\}_{w \in W}$  is a family of mappings  $v_w : For^{\circ\boxplus} \rightarrow \mathbf{2}$  (where  $For^{\circ\boxplus}$  is the set of formulas of **BDmbC**) satisfying clauses (i) - (vi) of Definition 3 plus the clauses below.

$$\text{(vii)} \quad v_w(\boxplus\alpha) = 1 \text{ iff } v_{w'}(\alpha) = 1 \text{ for every } w' \text{ in } W \text{ such that } wR^\circ w';$$

$$\text{(viii)} \quad v_w(\boxplus\neg\alpha) = 1 \text{ implies } v_{w'}(\alpha) = 0 \text{ for every } w' \text{ in } W \text{ such that } wR^\circ w'. \quad \blacksquare$$

Soundness of **BDmbC** with respect to Kripke structures is proved straightforwardly. The proof of completeness of **BDmbC** will be sketched below.

**Lemma 1** Let  $\Delta$  be a set  $\alpha$ -saturated in **BDmbC**, that is:  $\Delta \not\vdash_{\mathbf{BDmbC}} \alpha$  but  $\Delta, \psi \vdash_{\mathbf{BDmbC}} \alpha$  if  $\psi \notin \Delta$ . Then  $\Delta$  is a closed theory such that:

$$\text{(i)} \quad \beta \wedge \gamma \in \Delta \text{ iff } \beta \in \Delta \text{ and } \gamma \in \Delta;$$

$$\text{(ii)} \quad \beta \vee \gamma \in \Delta \text{ iff } \beta \in \Delta \text{ or } \gamma \in \Delta;$$

<sup>3</sup>A similar relationship is present between the physical and alethical necessity, cf. [5].

- (iii)  $\beta \Rightarrow \gamma \in \Delta$  iff  $\beta \notin \Delta$  or  $\gamma \in \Delta$ ;
- (iv)  $\beta \notin \Delta$  implies  $\neg\beta \in \Delta$ ;
- (v)  $\beta, \neg\beta \in \Delta$  implies  $\circ\beta \notin \Delta$ ;
- (vi)  $\boxminus\beta \notin \Delta$  or  $\boxminus\neg\beta \notin \Delta$ . ■

**Definition 7** Let  $\Delta$  be a set  $\alpha$ -saturated in **BDmbC**.

- (i) The  $\circ$ -denecessitation of  $\Delta$  is the set  $\circ Den(\Delta) =_{def} \{\beta : \circ\beta \in \Delta\}$ .
- (ii) The  $\boxminus$ -denecessitation of  $\Delta$  is the set  $\boxminus Den(\Delta) =_{def} \{\beta : \boxminus\beta \in \Delta\}$ . ■

**Lemma 2** Let  $\Delta$  be a  $\alpha$ -saturated set in **BDmbC**.

- (i) The sets  $\circ Den(\Delta)$  and  $\boxminus Den(\Delta)$  are closed theories of **BDmbC**.
- (ii)  $\boxminus Den(\Delta) \subseteq \circ Den(\Delta)$ .
- (iii)  $\circ\beta \notin \Delta$  implies  $\circ Den(\Delta), \neg\beta \not\vdash_{\mathbf{BDmbC}} \beta$ .
- (iv)  $\boxminus\beta \notin \Delta$  implies  $\boxminus Den(\Delta), \neg\beta \not\vdash_{\mathbf{BDmbC}} \beta$ . ■

**Definition 8** The *canonical model* for **BDmbC** is a tuple

$$\mathcal{M}_c = \langle \mathcal{W}, R, R^\circ, \{v_\Delta\}_{\Delta \in \mathcal{W}} \rangle$$

such that:

1.  $\mathcal{W} = \{\Delta \subseteq For^{\circ\boxminus} : \Delta \text{ is a } \alpha\text{-saturated set in } \mathbf{BDmbC} \text{ for some } \alpha\}$ ;
2.  $R = \{\langle \Delta, \Delta' \rangle \in \mathcal{W} \times \mathcal{W} : \circ Den(\Delta) \subseteq \Delta'\}$ ;
3.  $R^\circ = \{\langle \Delta, \Delta' \rangle \in \mathcal{W} \times \mathcal{W} : \boxminus Den(\Delta) \subseteq \Delta'\}$ ;
4.  $v_\Delta(\beta) = 1$  iff  $\beta \in \Delta$ . ■

**Proposition 1** The canonical model  $\mathcal{M}_c$  is a Kripke structure for **BDmbC**.

**Theorem 1 (Completeness for BDmbC)** Let  $\Gamma \cup \{\alpha\}$  be a set of formulas in  $For^{\circ\boxminus}$ . Then:  $\Gamma \vDash_{\mathbf{BDmbC}} \alpha$  implies  $\Gamma \vdash_{\mathbf{BDmbC}} \alpha$ .

**Prof** Suppose that  $\Gamma \not\vdash_{\mathbf{BDmbC}} \alpha$ . Using Theorem 56 given in [3] (which applies to a broad class of logics), we can extend  $\Gamma$  to a  $\alpha$ -saturated set  $\Delta$  in **BDmbC**. Let  $\mathcal{M}_c$  be the canonical model for **BDmbC**. Thus,  $\mathcal{M}_c$  is a Kripke structure for **BDmbC** and  $\Delta$  is a possible world of  $\mathcal{M}_c$  such that  $\mathcal{M}_c, \Delta \vDash_{\mathbf{BDmbC}} \Gamma$  and  $\mathcal{M}_c, \Delta \not\vDash_{\mathbf{BDmbC}} \alpha$ . Therefore,  $\Gamma \not\vdash_{\mathbf{BDmbC}} \alpha$ . ■

## 4 The Chisholm Paradox, revisited

The well-known Chisholm's Paradox, introduced in [7] (see also [2, 10, 12, 13]) can be obtained from the following four sentences (called *Chisholm's set*):

1. It ought to be that a certain man go to help his neighbors.

2. It ought to be that if he goes he tell them he is coming.
3. If he does not go, he ought not to tell them he is coming.
4. He does not go.

Let A be “a certain man go to help his neighbors” and let B be “he tell them he is coming”. Using **SDL** we have four ways to formalize this set of sentences:

$$\begin{aligned}\Gamma_1 &= \{\bigcirc A, \bigcirc(A \Rightarrow B), \bigcirc(\neg A \Rightarrow \neg B), \neg A\} \\ \Gamma_2 &= \{\bigcirc A, \bigcirc(A \Rightarrow B), \neg A \Rightarrow \bigcirc\neg B, \neg A\} \\ \Gamma_3 &= \{\bigcirc A, A \Rightarrow \bigcirc B, \bigcirc(\neg A \Rightarrow \neg B), \neg A\} \\ \Gamma_4 &= \{\bigcirc A, A \Rightarrow \bigcirc B, \neg A \Rightarrow \bigcirc\neg B, \neg A\}\end{aligned}$$

In  $\Gamma_1$  the sentence 3 is derived from the sentence 1; in  $\Gamma_4$  the sentence 2 is derived from the sentence 4; finally, in  $\Gamma_3$  sentences 2 and 3 are derived from sentences 4 and 1, respectively. Thus, the only possibility of obtaining a set of logically independent sentences is  $\Gamma_2$ . But  $\bigcirc A$  and  $\bigcirc\neg A$  follows from  $\Gamma_2$ , and so  $\Gamma_2$  is logically trivial. In other words, the formalization in **SDL** of Chisholm’s set produces a set of sentences which is either logically dependent or logically trivial, against the intuition.

Let us analyze the same situation, but now using the logic **BDmbC**. Now there are more ways to formalize Chisholm’s set, because the formal language is richer. For simplicity, consider the following notation:

**Definition 9**

$$\begin{aligned}\sim\alpha &\equiv_{df} \alpha \Rightarrow \perp_\alpha \text{ (strong negation);} \\ \mathcal{F}_0\alpha &\equiv_{df} \bigcirc\neg\alpha \text{ (very weak forbidden);} \\ \mathcal{F}_1\alpha &\equiv_{df} \bigcirc\sim\alpha \text{ (weak forbidden);} \\ \mathcal{F}_2\alpha &\equiv_{df} \Box\sim\alpha \text{ (strong forbidden).} \quad \blacksquare\end{aligned}$$

It is easy to see the following:  $\Box\sim\alpha$  is equivalent to  $\Box\neg\alpha$ ;  $\mathcal{F}_2\alpha$  implies  $\mathcal{F}_1\alpha$ , and  $\mathcal{F}_1\alpha$  implies  $\mathcal{F}_0\alpha$ . On the other hand  $\alpha \Rightarrow \beta$  follows from  $\sim\alpha$ , and  $\sim\alpha \Rightarrow \beta$  follows from  $\alpha$ . In order to shorten the presentation, let  $\oplus, \otimes, \odot \in \{\bigcirc, \Box\}$ ;  $\div, \wr \in \{\neg, \sim\}$  and  $i \in \{0, 1, 2\}$ . Then, taking into account the logical dependencies in **BDmbC** mentioned above, Chisholm’s set can be formalized in **BDmbC** in the following ways, without having logical dependencies:

$$\begin{aligned}\Gamma_{\oplus \otimes \odot \div \wr}^1 &= \{\oplus A, \otimes(A \Rightarrow B), \odot(\neg A \Rightarrow \div B), \wr A\} \\ \Gamma_{\oplus \otimes \div i \wr}^2 &= \{\oplus A, \otimes(A \Rightarrow B), \div A \Rightarrow \mathcal{F}_i B, \wr A\} \\ \Gamma_{\oplus \otimes \odot \div}^3 &= \{\oplus A, A \Rightarrow \otimes B, \odot(\neg A \Rightarrow \div B), \neg A\} \\ \Gamma_{\oplus \otimes \div i}^4 &= \{\oplus A, A \Rightarrow \otimes B, \div A \Rightarrow \mathcal{F}_i B, \neg A\}\end{aligned}$$

Of course some of the sets above are logically trivial in **BDmbC**. However, it is clear that the possibilities for translating Chisholm’s set into the logic **BDmbC** are ample, and several interesting logical consequences can be obtained in each non-trivial axiomatization. For instance, from  $\Gamma_{\oplus \oplus \odot \div \wr}^1$  it follows  $\oplus B$  but  $\odot \div B$  does not follow. By its turn,  $\Gamma_{\oplus \otimes \neg_i}^4$  produces  $\mathcal{F}_i B$  but  $\otimes B$  does not follow. Analogously, from  $\Gamma_{\oplus \otimes \oplus \div}^3$  it follows  $\oplus \div B$  but  $\otimes B$  does not follow. On the other hand,  $\Gamma_{\oplus \oplus \div_i \div}^2$  produces  $\oplus B$  and  $\mathcal{F}_i B$ . Thus, if  $i = 0$  we obtain contradictory obligations that do not trivialize.

In terms of Chisholm’s paradox, if we ask to **BDmbC** “The man should tell his neighbors he is coming?”, from  $\Gamma_{\oplus \oplus \odot \div \wr}^1$  the answer would be “YES”; from  $\Gamma_{\oplus \otimes \neg_i}^4$  or  $\Gamma_{\oplus \otimes \oplus \div}^3$  the answer would be “NO”; and from  $\Gamma_{\oplus \oplus \div_0 \div}^2$  the answer would be “YES and NO”. Of course the normative force of these answers depends upon the choice of the deontic operators in the formalizations above.

## 5 Conclusion

In this paper a solution to deontic paradoxes such as Chisholm’s Paradox was proposed by using **BDmbC**, a bimodal paraconsistent deontic logic. Specifically, the paradoxes are dissolved in two ways: (1) by allowing a richer repertoire of connectives which avoids logical dependencies, and (2) by avoiding logical collapse in the presence of contradictory obligations. Similar solutions can be obtained in weaker systems such as **DmbC** and **SDmbC**, although **BDmbC** provides more alternatives. It is worth noting that the former are very simple systems, with just one modal monadic operator, and not dyadic as the ones considered, for instance, in [2, 12, 13].

There are several perspectives of future research. As suggested in [8], the concept of **LDI**’s can be generalized to a wider class of paraconsistent logics, for instance those of [3]. On the other hand, system **BDmbC** opens the possibility of going further and create multimodal paraconsistent logics. Another line of research is the following: note that  $\Box \neg \alpha$  is equivalent to  $\Box \sim \alpha$  in **BDmbC**, as it was mentioned above. It would be interesting to consider an extension of **BDmbC** validating the equivalence between  $\Box \alpha$  and  $\Box \circ \alpha$ .

To conclude, we believe that the two systems herein introduced, as well as the new modal concepts here outlined, open interesting perspectives in the study and classification of **LDI**-systems, as well as its potential applications.

**Acknowledgements:** This research was financed by FAPESP (Brazil), Thematic Project ConsRel 2004/1407-2. Both authors were also supported by grants from the Brazilian Council of Research (CNPq).

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